



# Decision-making and control co-design for multi-agent systems: a hierarchical design methodology

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Decision-making and control are two of the foremost key ingredients in any autonomous intelligent system. Their co-design has been well-recognized even since the early days of control [1]. Recently, motivated by the wide applications of physical networked systems in different areas to cooperatively meet some cyber computation/communication objectives and constraints, there is an urgent need towards an efficient decision-making and control co-design for these cyber-physical systems. In such designs, we are required to determine distributed rules that can steer these physical plants to a steady-state corresponding to system-level decision-making problems. Nevertheless, the twisted complexities resulting from different aspects related to optimization, control, and computation become a great challenge to resolve such problems.

To achieve a decision-making and control co-design, the most direct idea might be to impose a time-scale separation property on the decision-making and control dynamics [2]. However, such treatments can be significantly inefficiency and even fail to guarantee the expected performance when the decision-making requirement and agent dynamics are tightly coupled. Thus, many efforts have been made to manage the decision-making dynamics and control dynamics at the same time for several special classes of tightly coupled multi-agent systems [3–6]. We call such design philosophy as a *tightly integrated architecture* since various complexities are simultaneously dealt with in this integrated method, which may be a good choice for simple co-design problems. For general co-design problems of complex multi-agent systems, it will

reveal its lack of flexibility and reusability. Consequently, the design flow is often ad hoc and highly relies on experience and post-design testing.

In the following, we envision a hierarchical design methodology to divide and conquer the problem complexities from different aspects and show how the considered decision-making and control co-design problem for complex multi-agent systems can be resolved in a hierarchical way. The design procedure can be summarized as follows.

*Step 1 (Preprocessing)* In this step, both the decision-making and agent dynamics will be transformed into some standard forms to facilitate further treatments. For example, we might recast the informal control goal into an unambiguous specification by suitable languages. We can also use local information to convert the heterogeneous multi-agent systems into normal forms. Such preprocessing suffices us to only focus on several canonical kinds of decision-making problem for multi-agent systems of certain specific structure.

*Step 2 (Optimal trajectory generation)* We construct an auxiliary multi-agent system for the physical multi-agent system under the name of optimal trajectory generator to satisfy the specification described in *Step 1*. The choice of auxiliary agent dynamics depends upon the given task. For example, suppose we focus on steering the physical multi-agent systems to some steady-state corresponding to a system-level objective, single integrators are good enough for our purpose. Since the auxiliary agent dynamics would be simple and perfectly known, we can leverage plenty of established decision-making algorithms to generate an optimal trajectory for each agent to fulfill the specification. Remarkably, the generated trajectory can be different depending upon the available information and used strategies.

*Step 3 (Reference tracking)* We consider the physical multi-agent system and construct tracking controllers for them with the generated trajectory in *Step 2* as a reference signal. Different from the first two steps, we do not have to care too much about the decision-making issue in this step. In fact, embedding the generator into the feedback loop,

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we are able to meet the specification in *Step 1* perhaps with some acceptable errors. The main focus is now the various physical constraints, uncertainties, and disturbances in agent dynamics. As such, the well-established control techniques and algorithms can be systemically utilized to handle these issues. In practice, *Step 2* and *Step 3* might be repeatedly and iteratively implemented to ensure the specification.

By contrast, we call this design philosophy as a *hierarchical architecture* since it naturally exhibits a two-layer hierarchy accounting for different aspects of the full co-design problem. At the upper layer or decision-making layer, we build an auxiliary multi-agent system to generate an optimal trajectory for the lower layer to meet the given specification. At the lower layer or control layer, we develop a reference tracking controller for each agent to follow the generated optimal trajectory. Bringing the two parts together, we will have the final algorithms serving to the decision-making and control co-design for considered multi-agent systems. Under this architecture, the full problem complexities from the decision-making task, complex agent dynamics, and incomplete information can be loosely decoupled. Thus, we only have to focus on several much simpler subproblems and their composition issue, which certainly simplifies the full co-design problem and may enable us to obtain more elaborate results.

It is interesting to remark that many existing decision-making and control co-design results fall into this hierarchical architecture confirming its flexibility. Lets take the optimization and control co-design problem for an example. Depending upon whether the control layer actively feeds information back to the decision-making layer, we can categorize these algorithms as off-line class or real-time class.

In off-line algorithms, the decision-making layer can be designed independently from the control layer to generate an optimal trajectory for each agent to track. Such class of algorithms are well suitable for the case when the local gradient function is analytically known by each agent. Distributed optimal coordination designs in [7,8] fall into the off-line class where an auxiliary double-integrator network was built to generate the optimal trajectory. Meanwhile, it was shown in [9] that single-integrator auxiliary multi-agent systems are good enough for the decision-making layer in such co-designs. Thus, many existing distributed optimization rules can be directly utilized in the decision-making layer. Since the optimization dynamics and control dynamics are now in a cascading form, the overall performance analysis can be easily complete. Thus, we are allowed to pay special attention to extra design features, e.g., model uncertainties and finite-time convergence [10,11].

When the measured gradient corresponding to real-time agent output is only available, the off-line algorithms fail to be implemented and we have to resort to real-time distributed algorithms. In this case, there exists an extra channel from

the control layer to the decision-making layer to feed the real-time gradients during the control process back to the decision-making layer. That is, the optimization dynamics and control dynamics are now in an interconnected form. Thus, some ideas from nonlinear control systems, such as input-to-state stability and small-gain theorem, are usually required to ensure the convergence of the full algorithm. For example, the three-step method was given to achieve the optimal output consensus for linear multi-agent systems in [9], and recent research efforts have developed real-time algorithms for different classes of multi-agent systems [12–14].

It is also worthwhile to point out that the hierarchical architecture naturally implies a high reusability of each individual layer/module. For example, the optimal trajectory generator developed in [10] has also been utilized to resolve the co-design problem for other different classes of multi-agent systems [15]. On the other hand, when the optimization requirement is different, we only have to change another generator correspondingly while using the similar conventional tracking controllers as shown in [16–19].

In this letter, we briefly introduce some representative approaches in the decision-making and control co-design for multi-agent systems. Particularly, we emphasize on a hierarchical design methodology that can efficiently manage the problem complexities in a modular fashion. We believe that this methodology is very promising and will play a crucial role in the future research of decision-making and control co-design for complex multi-agent systems. At this stage, many fundamental problems remain unsolved. For example, compared with off-line designs, the more practical real-time algorithms are relative few and deserve further attention. Meanwhile, most of existing results are derived for continuous-time decision-making and agent dynamics. It is interesting to further exploit the method to handle general discrete-time and even hybrid multi-agent systems. Yet another interesting direction is to extend the hierarchical design methodology to tackle other complex decision-making problems, e.g., Nash equilibrium seeking [20].

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