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Leader-following coordination problem with an uncertain leader in a multi-agent system

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Abstract: A leader-following coordination problem is considered in this study where the active leader is driven by an unknown input. With the help of output regulation and disturbance decoupling, a distributed protocol is proposed to solve this problem. Two cases are studied based on whether the bound of uncertain (disturbance) inputs of the leader is known. When the bound is unknown, an adaptive technique is employed to achieve the goal. Simulations are also given to verify the effectiveness of these controllers.

1 Introduction

The past decade has witnessed a rapid development in the field of multi-agent system and fruitful results have been achieved. As one of the important problems, the leader-following coordination problem of multi-agent system has been studied from different viewpoints. The basic idea of the leader-following framework is to select one or more leaders based on given tasks, and lead the whole group of agents to achieve our goals. Jadbabaie *et al.* [1] considered such a leader-following consensus problem and proved that if all the agents were jointly connected with their leader, their states would converge to that of the leader. Then many results were obtained on consensus of leader-following multi-agent systems with agents in the form single-integrators or double integrators in [2]. Hong *et al.* [3], proposed a distributed observer-based control law using local information to track an integrator-type leader, whereas the target aggregation was studied when a leader becomes a convex set [4]. Leader-following consensus problem of general linear system was also studied in [5] under switching topologies. Recently, a general framework based on output regulation theory [6–8] with multi-agent system has been investigated [9–11], distributed protocols were proposed to achieve both references tracking and disturbances rejection.

However, all those results were obtained based on the assumption that the dynamic of the leader is exactly known, that is, the input of the leader is either equal to zero or available to all the followers. Sometimes, it may be restrictive or unpractical, especially when the leader is an uncooperative target.

A distributed tracking problem was investigated in the absence of velocity or acceleration of measurements in [12] when an agent is a particle moving under Newton's law. Owing to the non-zero control input, a variable structure controller was proposed for the followers. Moreover, Li *et al.* [13] extended it to the case of common linear agents using the same method. However, they assumed the leader and followers have the same system matrices.

In this paper, we consider this leader-following problem with an uncertain leader, whose dynamics is totally different from those of the followers. Two different protocols including static and dynamical rules are proposed to solve this problem. Note that we consider the leader-following problem when the leader and followers are different linear systems and the leader contains uncertain inputs. The contribution of the work is at least 2-fold:

- This work can be viewed as an extension of well-studied consensus problem (including state or output consensus). When there are no uncertainties in the leader, our results become consistent with the cooperative output regulation results considered in [2, 10].
- Distributed controllers are proposed to deal with the uncertain inputs in the leader under some conditions combined with classical disturbance decoupling problem. When the leader and followers share the same dynamics, it includes the results in [13] as a special case.

The rest of this paper is organised as follows. In Section 2, some preliminaries and problem formulation are given. Then our main results are presented in Section 3, where two types of control laws are given. Several examples are discussed in Section 4. Finally, simulations and our conclusion remarks are presented.

2 Preliminaries and formulation

First of all, we introduce some basic concepts and notations in graph theory (referring to [14] for details). A directed graph (or digraph) is usually denoted as $\mathcal{G}=(\mathcal{N},\mathcal{E})$, where $\mathcal{N}=\{1,2,\ldots,n\}$ is the set of nodes and $\mathcal{E}$ is the set of arcs, each element of which is an ordered pair of distinct nodes in $\mathcal{N}$. (i,j) denotes an arc leaving from node i and entering node j. A walk in digraph $\mathcal{G}$ is an alternating sequence $i_1e_1i_2e_2\cdots e_{k-1}i_k$ of nodes i_l and arcs $e_m=(i_m,i_{m+1})\in\mathcal{E}$ for $l=1,2,\ldots,k$. If there exists a walk from node i to node j then node j is said to be reachable from node i. In particular, each node is thought to be reachable by itself. A node that is reachable from any node of $\mathcal{G}$ is called a globally reachable node of $\mathcal{G}$.

Here, we consider a system consisting of m agents and a leader (denoted as node 0). In the sequel, the state of agent i is denoted by x_i for $i=1,\ldots,m$. With regarding the m agents as the vertices in $\mathcal{V}$, the relationships between n agents can be conveniently described by a simple and graph $\mathcal{G}$, where (i,j) defines one of the graph's arcs if agent j is reachable from agent j. N_i denotes the set of labels of those agents, from which agent i $(i=1,\ldots,m)$ can be reachable at time t. The weighted adjacency matrix of $\mathcal{G}$ is denoted by $A_m=[a_{ij}]\in\mathcal{R}^{m\times m}$, where $a_{ii}=0$ and $a_{ij}\geq 0$ ($a_{ij}>0$ if there is a walk from agent i to agent j). Its degree matrix $D_m=\text{diag}\{d_1,\ldots,d_m\}\in\mathcal{R}^{m\times m}$ is a diagonal matrix, where diagonal elements $d_i=\sum_{j=1}^{m}a_{ij}$ for $i=1,\ldots,m$. Then the Laplacian of the graph is defined as $L=D_m-A_m$.

In what follows, we mainly concern another graph $\bar{\mathcal{G}}$ associated with the system consisting of m agents and the leader. In fact, $\bar{\mathcal{G}}$ contains m agents (related to graph $\mathcal{G}$) and the leader with directed edges from some agents to the leader by the connection weights $a_{i0}>0$ (if agent i is connected to the leader, or equivalently, agent i is a neighbor of the leader, denoted by $i\in N_0$).

A_0 is an $m\times m$ diagonal matrix whose ith diagonal element is a_{i0} (if agent i is connected to the leader or node 0, namely $i\in N_0$). With L still representing the Laplacian matrix of the digraph $\mathcal{G}$, we define a matrix $H=L+A_0$ to describe the connectivity of the whole graph $\bar{\mathcal{G}}$.

Lemma 1 [3]: For any undirected graph $\mathcal{G}$, H is positive definite if and only if node 0 of $\bar{\mathcal{G}}$ is globally reachable.

Node 0 in $\bar{\mathcal{G}}$ should be globally reachable to make the information of leader is reachable, and therefore at least one agent in each component of $\mathcal{G}$ is connected to the abstract model or the leader. Otherwise, the coordination between the agents and the leader cannot be achieved. Hence, the following assumption is standard on the connectivity of the considered multi-agent system.

Assumption 1: Node 0 is always globally reachable in $\bar{\mathcal{G}}$.

Then we formulate our problem as follows. Consider a group of N agents and one leader. N followers are described by

$$\begin{cases}\dot{x}_i=Ax_i+Bu_i\\ y_i=Cx_i\quad (i=1,\ldots,N)\end{cases}\tag{1}$$

where $A\in\mathcal{R}^{m\times m},B\in\mathcal{R}^{m\times n},C\in\mathcal{R}^{p\times m}$ and (A,B) is stabilisable. x_i, y_i and u_i are the state variable, output and control input of agent i, respectively.

The leader is described as follows

$$\begin{cases}\dot{z}=Sz+Dv\\ y_0=Fz\end{cases}\tag{2}$$

where $S\in\mathcal{R}^{q\times q},D\in\mathcal{R}^{q\times r},F\in\mathcal{R}^{p\times q}$ with z, y_0 as its state variable and output variable, respectively. Without loss of generality, we assume D is full-ranked in column. v is an unstructured but bounded uncertainty. In fact, it may be its control input but unavailable to any follower. Similar problems have been investigated by some authors. For example, when $S=0$, $D=1$ or $S=0_{2\times 2},D=[1,0]^{\text{T}}$, it comes to be the case considered in [12] in the absence of velocity or acceleration of measurements of the leader, while v was deemed as some bounded disturbances in [3]. When $S=A$, $D=B$, this problem was also investigated by Li *et al.* [13].

We aim to construct distributed controllers for the followers to drive all $e_i=y_i-y_0(i=1,\ldots,N)$ to approach zeros.

In the case when there is no uncertain leader input, this leader-following coordination problem has been studied as a distributed output regulation problem by taking the leader as an exosystem and e_i as the distributed regulated variable, and fruitful results have be achieved. However, in practice, the assumption that the exosystem is exactly known may be restrictive.

Unlike the distributed output regulation problem discussed in [9, 10, 15], we consider a general case when v is time-varying and unknown with the following wild assumptions:

Assumption 2: v is continuous and bounded, that is, $\|v\|_\infty<\gamma$, where γ is a positive constant.

Apparently, when $v=0$, this problem includes the distributed output regulation [9, 10, 15] as special cases. Here, we cannot use any information of the uncertainties v but its bound, although it will be showed later that the assumption that knowing the bound of v can also be relaxed.

If there exists a state feedback control law $u=u(x_i,x_j,z)(j\in\mathcal{N}_i)$, such that $e_i(t)\to 0$ when $t\to\infty$ for any initial conditions $x_i(0)$ and $z(0)$ $(i=1,\ldots,N)$, we say this leader-following coordination problem is solved by a full-information controller.

As that in distributed output regulation problem [10], we give a standard assumption,

Assumption 3: There exist two matrices X and U such that the following linear matrix equations hold

$$\begin{cases}XS=AX+BU\\ CX=F\end{cases}\tag{3}$$

Remark 1: The above Sylvester equation has been showed to be necessary and widely employed when solving output regulation problem. When $S=A$, $D=B$, $F=C$, it holds naturally with $X=I,U=0$. Its special variant

$$BU=S-A$$

is a key ingredient to solve a leader-following consensus problem [10]. A sufficient and checkable condition to assure the solvability of these equations is often mentioned as

$$\text{rank}\left(\begin{bmatrix}A-\lambda I & B\\ C & 0\end{bmatrix}\right)=n+p$$

for all $\lambda\in\sigma(S)$.

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To deal with the uncertainties about the leader's input, we need another assumption. Since there must be constant matrices T_1 and E with proper dimensions such that $BT_1 + E = I_m$, and then $XDv = (BT_1 + E)XDv = BT_1XDv + EXDv$. We denote $BT_1XDv \triangleq \phi_m$ and $EXDv \triangleq \phi_n$. The following is inspired by [16] for generalised output regulation.

Assumption 4: $\text{Im}(EXD) \in \mathcal{V}^-(A, B, C)$, where $\text{Im}(EXD)$ is the column space of the matrix EXD, $\mathcal{V}^-(A, B, C)$ is the largest subspace $\mathcal{V}$ of $\mathcal{R}^n$ for which there exists a matrix K such that $\mathcal{V}$ is $(A + BK)$-invariant and contained in $\text{Ker}(C) \triangleq \{\xi \in R^m \mid C\xi = 0\}$ and such that the eigenvalues of $(A + BK)|_{\mathcal{V}}$ are on the open left-half complex plane.

Remark 2: This assumption can assure $EXDv$ is decoupled from the output of composite system. Unlike the condition needed in [16], Assumption 4 only needs that a part of the original disturbances can be decoupled from the output of composite system, and less restrictive than the one used in [16]. In fact, we will also show that this assumption will hold even when the constraint on XD as in [16] is not satisfied.

3 Main results

In this section, we should design distributed controllers for agents and solve this leader-following coordination problem.

Since the leader may not be directly connected with all followers, we propose the following distributed observer for agent i

$$\dot{z}_i = Sz_i - \mu\left[\sum a_{ij}(z_i - z_j) + a_{i0}(z_i - z)\right] + q_i \tag{4}$$

where q_i will be designed later.

Under Assumption 4, there is K such that $EXDv$ is decoupled from the output of composite system, and then a distributed protocol for output regulation can be given in the following form

$$\begin{aligned} u_i &= K(x_i - Xz_i) + Uz_i + p_i \\ \dot{z}_i &= Sz_i - \mu\left[\sum a_{ij}(z_i - z_j) + a_{i0}(z_i - z) + q_i\right] \end{aligned}$$

Denoting $K_1 = K$, $K_2 = U - KX$, we have

$$\begin{aligned} u_i &= K_1x_i + K_2z_i + p_i \\ \dot{z}_i &= Sz_i - \mu\left[\sum a_{ij}(z_i - z_j) + a_{i0}(z_i - z)\right] + q_i \end{aligned} \tag{5}$$

Remark 3: Notice that, when p_i and q_i are omitted, this control law solves the distributed output regulation problem with the same followers but with $v = 0$ in the exosystem [10]. In other words, p_i and q_i are designed to deal with the uncertain parts v. We assume $D \neq 0$ without loss of generality.

In the following two subsections, we construct p_i and q_i considering two circumstances: when γ is known or unknown.

3.1 γ is known

Assume γ, the bound of v, is known in this subsection. We first introduce the following lemma, which establishes the convergency of the distributed observer (4) under a designed q_i.

Let $\bar{z}_i = z_i - z$. Then

$$\dot{\bar{z}}_i = S\bar{z}_i - \mu\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\bar{z}_i - \bar{z}_j) + a_{i0}\bar{z}_i\right] + q_i - Dv$$

or in a compact form

$$\dot{\bar{z}} = (I \otimes S - \mu H \otimes I)\bar{z} + q - \mathbf{1} \otimes (Dv) \tag{6}$$

We can obtain the following lemma

Lemma 2: Under Assumption 1, if

$$q_i = -\gamma D\text{sgn}\left(D^{\mathrm{T}}\sum_{j\in\mathcal{N}_i} a_{ij}(z_i - z_j) + a_{i0}(z_i - z)\right)$$

then the distributed observer will asymptotically converge to z for a large enough μ.

Proof: Consider the error system in the following

$$\dot{\bar{z}} = (I \otimes S - \mu H \otimes I)\bar{z} + q - \mathbf{1} \otimes (Dv) \tag{7}$$

Take a Lyapunov function as $V = \bar{z}^{\mathrm{T}}(H \otimes I)\bar{z}$, and its derivative is as follows

$$\begin{aligned} \dot{V} &= \bar{z}^{\mathrm{T}}(H \otimes I)\{[I \otimes S - \mu H \otimes I]\bar{z} + q - \mathbf{1} \otimes (Dv)\} \\ &\quad + \{[I \otimes S - \mu H \otimes I]\bar{z} + q - \mathbf{1} \otimes (Dv)\}^{\mathrm{T}}(H \otimes I)\bar{z} \\ &= \bar{z}^{\mathrm{T}}[H \otimes (S + S^{\mathrm{T}}) - 2\mu H^2 \otimes I]\bar{z} \\ &\quad + 2\bar{z}^{\mathrm{T}}(H \otimes I)[q - \mathbf{1} \otimes (Dv)] \end{aligned}$$

Since H is positive, there exists an unitary matrix U such that $\Lambda = U^{\mathrm{T}}HU$ is in diagonal form, and let $\tilde{z} = (U \otimes I)\bar{z}$,

$$\begin{aligned} &\bar{z}^{\mathrm{T}}[H \otimes (S + S^{\mathrm{T}}) - 2H^2 \otimes I]\bar{z} \\ &\quad = \tilde{z}^{\mathrm{T}}[\Lambda \otimes (S + S^{\mathrm{T}}) - 2\Lambda^2 \otimes I]\tilde{z} \\ &\quad = \sum \lambda_i\tilde{z}_i^{\mathrm{T}}(S + S^{\mathrm{T}} - 2\mu\lambda_i I)\tilde{z}_i \end{aligned}$$

We can select a large $\mu > 0$ such that for all i,

$$S + S^{\mathrm{T}} - 2\mu\lambda_i I < -\max_{j=1,\ldots,N}(1/\lambda_j)I$$

In fact, since $S + S^{\mathrm{T}}$ is symmetric, it is enough to take a $\mu > 0$ such that $(2\mu\lambda_i + \max_{j=1,\ldots,N}(1/\lambda_j))I - S - S^{\mathrm{T}}$ is strictly diagonally dominant.

Hence, we can obtain

$$\bar{z}^{\mathrm{T}}[H \otimes (S + S^{\mathrm{T}}) - 2\mu H^2 \otimes I]\bar{z} < -\bar{z}^{\mathrm{T}}\bar{z}$$

Then we consider the last term $2\bar{z}^{\mathrm{T}}(H \otimes I)[q - \mathbf{1} \otimes (Dv)]$, and we can find

$$q = -\gamma(I \otimes D)\mathrm{sgn}[(H \otimes D^{\mathrm{T}})\bar{z}]$$

Then

$$\begin{aligned} &2\bar{z}^{\mathrm{T}}(H \otimes I)q - 2\bar{z}^{\mathrm{T}}(H \otimes I)(\mathbf{1} \otimes Dv) \\ &\quad = 2\bar{z}^{\mathrm{T}}(H \otimes I)q - 2\bar{z}^{\mathrm{T}}(H \otimes I)(I \otimes D)(\mathbf{1} \otimes v) \\ &\quad \le -2\gamma\|(H \otimes D^{\mathrm{T}})\bar{z}\|_1 + 2\gamma\|(H \otimes D^{\mathrm{T}})\bar{z}\|_1 \\ &\quad = 0 \end{aligned}$$

Hence

$$\dot{V} \le -\bar{z}^{\mathrm{T}}\bar{z}$$

It is easy to find that V and then $\bar{z}$ will asymptotically converge to zero. □

Clearly, since $A + BK_1$ is Hurwitz, there exist two positive definite matrices P and Q such that

$$(A + BK_1)^{\mathrm{T}}P + P(A + BK_1) = -Q \tag{8}$$

It is time to give the first main theorem.

Theorem 1: Under Assumptions 1–4, the leader-following coordination problem of agents described by (1) with the leader (2) can be solved by the control law (5) with

$$p_i = -\gamma\|T_1XD\|_2\mathrm{sgn} \times \left(B^{\mathrm{T}}P\left[\sum_{j\in\mathcal{N}_i} a_{ij}(x_i - x_j) + a_{i0}(x_i - Xz)\right]\right)$$

and

$$q_i = -\gamma D\mathrm{sgn}\left(D^{\mathrm{T}}\left[\sum_{j\in\mathcal{N}_i} a_{ij}(z_i - z_j) + a_{i0}(z_i - z)\right]\right)$$

Proof: With denoting $\bar{x}_i = x_i - Xz$ and $\bar{x} = [\bar{x}_1^{\mathrm{T}}, \ldots, \bar{x}_n^{\mathrm{T}}]^{\mathrm{T}}$, the closed-loop system is

$$\begin{aligned} \dot{\bar{x}} &= [I \otimes (A + BK_1)]\bar{x} + [I \otimes (BK_2)]\bar{z} \\ &\quad + (I \otimes B)p - (I \otimes XD)(\mathbf{1} \otimes v) \end{aligned}$$

$$\dot{\bar{z}} = [I \otimes S - \mu H \otimes I]\bar{z} + q - (\mathbf{1} \otimes D)(\mathbf{1} \otimes v)$$

Since $XDv = \phi_m + \phi_n$, recalling Assumption 4, we obtain

$$\begin{aligned} e_i &= C\bar{x}_i \\ &= Ce^{(t-t_0)(A+BK_1)}\bar{x}_i(t_0) \\ &\quad + C\int_{t_0}^{t} e^{-(t-\tau)(A+BK_1)}B(p_i - \phi_m)\,\mathrm{d}\tau \\ &\quad - C\int_{t_0}^{t} e^{-(t-\tau)(A+BK_1)}\phi_n \mathrm{d}\tau \\ &\quad + C\int_{t_0}^{t} e^{-(t-\tau)(A+BK_1)}BK_2\bar{z}_i\tau \\ &= Ce^{(t-t_0)(A+BK_1)}\bar{x}_i(t_0) \\ &\quad + C\int_{t_0}^{t} e^{-(t-\tau)(A+BK_1)}B(p_i - \phi_m)\,\mathrm{d}\tau \\ &\quad + C\int_{t_0}^{t} e^{-(t-\tau)(A+BK_1)}BK_2\bar{z}_i\,\mathrm{d}\tau \end{aligned}$$

that is, ϕ_n can be decoupled from e_i.

Hence, it is enough to prove that the following system is asymptotically stable under p_i and q_i

$$\begin{aligned} \dot{\bar{x}} &= [I \otimes (A + BK_1)]\bar{x} + (I \otimes BK_2)\bar{z} \\ &\quad + (I \otimes B)[p - (I \otimes T_1XD)(\mathbf{1} \otimes v)] \\ \dot{\bar{z}} &= (I \otimes S - \mu H \otimes I)\bar{z} + q - \mathbf{1} \otimes (Dv) \end{aligned}$$

As showed in Lemma 2, the second system will asymptotically converge to be zero, and we only have to show that the first system is input-to-state stable with $\bar{z}$ as its input.

Since $p_i = \lambda\mathrm{sgn}(B^{\mathrm{T}}P[\sum_{j\in\mathcal{N}_i}(x_i - x_j) + a_{i0}(x_i - Xz)])$ and $\lambda = -\gamma\|T_1XD\|_2$, the first subsystem is as follows

$$\begin{aligned} \dot{\bar{x}} &= [I \otimes (A + BK_1)]\bar{x} + (I \otimes BK_2)\bar{z} \\ &\quad + (I \otimes B)[\lambda\mathrm{sgn}[(H \otimes B^{\mathrm{T}}P)\bar{x}] \\ &\quad - (I \otimes T_1XD)(\mathbf{1} \otimes v)] \end{aligned}$$

Taking a Lyapunov function as

$$V = \bar{x}^{\mathrm{T}}(H \otimes P)\bar{x}$$

we consider its derivative (see equation at the bottom of the page)

$$\begin{aligned} \dot{V} &= ([I \otimes (A + BK_1)]\bar{x} + (I \otimes B)[p - (I \otimes T_1XD)(\mathbf{1} \otimes v)])^{\mathrm{T}}(H \otimes P)\bar{x} \\ &\quad + \bar{x}^{\mathrm{T}}(H \otimes P)([I \otimes (A + BK_1)]\bar{x} + (I \otimes B)[p - (I \otimes T_1XD)(\mathbf{1} \otimes v)]) \\ &\quad + 2\bar{x}^{\mathrm{T}}(H \otimes P)(I \otimes BK_2)\bar{z} \\ &= -\bar{x}^{\mathrm{T}}(H \otimes Q)\bar{x} + 2\bar{x}^{\mathrm{T}}(H \otimes PB)[p - (I \otimes T_1XD)(\mathbf{1} \otimes v)] + 2\bar{x}^{\mathrm{T}}(H \otimes P)(I \otimes BK_2)\bar{z} \\ &\le -\bar{x}^{\mathrm{T}}(H \otimes Q)\bar{x} + 2\lambda\bar{x}^{\mathrm{T}}(H \otimes PB)\,\mathrm{sgn}((H \otimes B^{\mathrm{T}}P)\bar{x}) \\ &\quad - 2\bar{x}^{\mathrm{T}}(H \otimes PB)(I \otimes T_1XD)(\mathbf{1} \otimes v) + 2\bar{x}^{\mathrm{T}}(H \otimes P)(I \otimes BK_2)\bar{z} \\ &\le -\bar{x}^{\mathrm{T}}(H \otimes Q)\bar{x} + 2\lambda\|(H \otimes B^{\mathrm{T}}P)\bar{x}\|_1 - 2\lambda\|(H \otimes B^{\mathrm{T}}P)\bar{x}\|_2 + 2\bar{x}^{\mathrm{T}}(H \otimes P)(I \otimes BK_2)\bar{z} \\ &\le -\bar{x}^{\mathrm{T}}(H \otimes Q)\bar{x} + 2\bar{x}^{\mathrm{T}}(H \otimes PBK_2)\bar{z} \end{aligned}$$

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Denoting $W(\bar{x}) \triangleq \bar{x}^{\mathrm{T}}(H \otimes Q)\bar{x}$, it follows

$$\dot{V} \leq W(\bar{x}) + 2\bar{x}^{\mathrm{T}}(H \otimes PBK_2)\bar{z}$$

where $W(\bar{x})$ is positive definite.

It is sufficient to conclude that this subsystem is input-to-state stable with $\bar{z}$. Recalling Lemma 4, we conclude that $\bar{x}$ and hence e will converge to be zero when $t \to \infty$. □

Remark 4: For an important special case when $S = A$, $D = B$, $F = C$, we can find that Assumptions 3 and 4 holds naturally in this circumstance $X = I$, $U = 0$ and with $\phi_n = 0$. Simplified static controllers can be given without using distributed observers as

$$u_i = -\max\left\{\frac{1}{\bar{\lambda}}, 1\right\} B^{\mathrm{T}}P\left[\sum_{j\in\mathcal{N}_I} a_{ij}(x_i - x_j) + a_{i0}(x_i - z)\right] + \lambda\mathrm{sign}\left(B^{\mathrm{T}}P\left[\sum_{j\in\mathcal{N}_I} a_{ij}(x_i - x_j) + a_{i0}(x_i - z)\right]\right)$$

where P is the positive definite solution of $A^{\mathrm{T}}P + PA + I - PBB^{\mathrm{T}}P = 0$ and $\bar{\lambda}$ is the minimal eigenvalue of H. In this case, it is consistent with the results obtained in [13].

3.2 γ is unknown

Here, we consider when v is bounded without a known γ.

To overcome the difficulty of unknown γ, we propose the following controller for agent i with a dynamic gain (see (9))

where θ_i is an updated gain and μ is a positive constant to be designed.

Then the following result can be obtained.

Theorem 2: Under Assumptions 1–4, the leader-following coordination problem of agents described by (1) with exosystem (2) can be solved by the control law (9).

Proof: With denoting $\bar{x}_i = x_i - Xz$ and $\bar{x} = [\bar{x}_1^{\mathrm{T}}, \ldots, \bar{x}_n^{\mathrm{T}}]^{\mathrm{T}}$, the closed-loop system can be written as

$$\begin{aligned}\dot{\bar{x}} &= [I \otimes (A + BK_1)]\bar{x} + [I \otimes (BK_2)]\bar{z} \\ &\quad + (I \otimes B)p - (I \otimes XD)(\mathbf{1} \otimes v) \\ \dot{\bar{z}} &= [I \otimes S - \mu H \otimes I]\bar{z} + q - (1 \otimes D)(\mathbf{1} \otimes v)\end{aligned}$$

Similarly, with Assumption 4, it is enough to prove the following system is asymptotically stable under these protocols

$$\begin{aligned}\dot{\bar{x}} &= [I \otimes (A + BK_1)]\bar{x} + (I \otimes BK_2)\bar{z} \\ &\quad + (I \otimes B)[p - (I \otimes T_1XD)(1 \otimes v)] \\ \dot{\bar{z}} &= [I \otimes S - \mu H \otimes I]\bar{z} + q - (\mathbf{1} \otimes D)(1 \otimes v)\end{aligned}$$

Under the control law (9), the closed-loop system is expressed as follows (see equation at the bottom of the page)

Based on the Lyapunov equation (8), take a Lyapunov function as

$$V = \bar{x}^{\mathrm{T}}(H \otimes P)\bar{x} + \bar{z}^{\mathrm{T}}(H \otimes I)\bar{z} + \sum(\theta_i - \alpha)^2$$

where α is a constant, which can be taken large than γ.

Considering its derivative

$$\begin{aligned}\dot{V} &= \bar{x}^{\mathrm{T}}(H \otimes [P(A + BK_1) + (A + BK_1)^{\mathrm{T}}P])\bar{x} \\ &\quad + 2\bar{x}^{\mathrm{T}}(H \otimes PBK_2)\bar{z} \\ &\quad + 2\bar{x}^{\mathrm{T}}(H \otimes PB)[p - (I \otimes T_1XD)(\mathbf{1} \otimes v)] \\ &\quad + \bar{z}^{\mathrm{T}}[H \otimes (S + S^{\mathrm{T}}) - 2\mu H^2 \otimes (I)]\bar{z}^{\mathrm{T}} \\ &\quad + 2\bar{z}^{\mathrm{T}}(H \otimes I)[q - \mathbf{1} \otimes (Dv)] + 2\sum(\theta_i - \alpha)\dot{\theta}_i\end{aligned}$$

$$\begin{aligned}u_i &= K_1x_i + K_2z_i - \theta_i\|T_1XD\|_2 \ \mathrm{sgn}\left(B^{\mathrm{T}}P\left[\sum_{j\in\mathcal{N}_i} a_{ij}(x_i - x_j) + a_{i0}(x_i - Xz)\right]\right) \\ \dot{z}_i &= Sz_i - \mu\left[\sum_{j\in\mathcal{N}_i} a_{ij}(z_i - z_j) + a_{i0}(z_i - z)\right] - \theta_i D\mathrm{sgn}\left(D^{\mathrm{T}}\left[\sum_{j\in\mathcal{N}_i} a_{ij}(z_i - z_j) + a_{i0}(z_i - z)\right]\right) \\ \dot{\theta}_i &= \|T_1XD\|_2\left\|B^{\mathrm{T}}P\left[\sum_{j\in\mathcal{N}_i} a_{ij}(x_i - x_j) + a_{i0}(x_i - Xz)\right]\right\|_1 + \left\|D^{\mathrm{T}}\left[\sum_{j\in\mathcal{N}_i} a_{ij}(z_i - z_j) + a_{i0}(z_i - z)\right]\right\|_1\end{aligned} \quad (9)$$

$$\begin{aligned}\dot{\bar{x}} &= [I \otimes (A + BK_1)]\bar{x} + (I \otimes BK_2)\bar{z} \\ &\quad + (I \otimes B)(-\theta\|T_1XD\|_2\mathrm{sgn}[(H \otimes B^{\mathrm{T}}P)\bar{x}] - (I \otimes T_1XD)(1 \otimes v)) \\ \dot{\bar{z}} &= [I \otimes S - \mu H \otimes I]\bar{z} - (I \otimes D)\theta\mathrm{sgn}((H \otimes D^{\mathrm{T}})\bar{z}) - (I \otimes D)(\mathbf{1} \otimes v) \\ \dot{\theta}_i &= \|T_1XD\|_2\left\|B^{\mathrm{T}}P\left[\sum_{j\in\mathcal{N}_i} a_{ij}(x_i - x_j) + a_{i0}(x_i - Xz)\right]\right\|_1 \\ &\quad + \left\|D^{\mathrm{T}}\left[\sum_{j\in\mathcal{N}_i} a_{ij}(z_i - z_j) + a_{i0}(z_i - z)\right]\right\|_1 \quad (i = 1, \ldots, N)\end{aligned}$$

From the proof of Lemma 4 and Theorem 1, we have (see equation at the bottom of the page)

and (see equation at the bottom of the page)

As a result (see equation at the bottom of the page)

By Young's inequality

$$\dot{V} \le -\bar{x}^{\mathrm{T}}(H \otimes Q)\bar{x} + \bar{z}^{\mathrm{T}}[H \otimes (S + S^{\mathrm{T}}) - 2\mu H^2 \otimes (I)]\bar{z}^{\mathrm{T}} + l^2\|\bar{x}\|_2 + \|(H \otimes PBK_2)\|_2^2/l^2\|\bar{z}\|_2$$

and we can take a constant l small enough so that

$$-\frac{1}{2}\bar{x}^{\mathrm{T}}(H \otimes Q)\bar{x} + l^2\bar{x}^{\mathrm{T}}\bar{x} \le 0$$

Then, with the fixed l, take a μ large enough such that

$$\bar{z}^{\mathrm{T}}[H \otimes (S + S^{\mathrm{T}}) - 2\mu H^2 \otimes (I)]\bar{z}^{\mathrm{T}} + \|(H \otimes PBK_2)\|_2^2/l^2\|\bar{z}\|_2 \le -\bar{z}^{\mathrm{T}}\bar{z}$$

Hence, we can obtain that

$$\dot{V} \le -\frac{1}{2}\bar{x}^{\mathrm{T}}(H \otimes Q)\bar{x} - \bar{z}^{\mathrm{T}}\bar{z} \le -\frac{\bar{\lambda}\sigma_{\min}(Q)}{2}\bar{x}^{\mathrm{T}}\bar{x} - \bar{z}^{\mathrm{T}}\bar{z} \triangleq W(\bar{x}, \bar{z})$$

Denoting $W(\bar{x}, \bar{z}) \triangleq \frac{\bar{\lambda}\sigma_{\min}(Q)}{2}\bar{x}^{\mathrm{T}}\bar{x} + \bar{z}^{\mathrm{T}}\bar{z}$, it follows

$$\dot{V} \le -W(\bar{x}, \bar{z})$$

$$\begin{aligned}
&\bar{x}(H \otimes [P(A + BK_1) + (A + BK_1)^{\mathrm{T}}P])\bar{x} + 2\bar{x}^{\mathrm{T}}(H \otimes PB)[p - (I \otimes T_1XD)(\mathbf{1} \otimes v)] \\
&\quad = -\bar{x}^{\mathrm{T}}(H \otimes Q)\bar{x} + 2\lambda\bar{x}^{\mathrm{T}}(H \otimes PB)\,\mathrm{sgn}((H \otimes B^{\mathrm{T}}P)\bar{x}) - 2\bar{x}^{\mathrm{T}}(H \otimes PB)(I \otimes T_1XD)(\mathbf{1} \otimes v) \\
&\qquad + 2\sum_i(\gamma - \theta_i)\|T_1XD\|_2\left\|B^{\mathrm{T}}P\left[\sum_{j\in\mathcal{N}_i}a_{ij}(x_i - x_j) + a_{i0}(x_i - Xz)\right]\right\|_1 \\
&\quad \le -\bar{x}^{\mathrm{T}}(H \otimes Q)\bar{x} + 2\sum_i(\gamma - \theta_i)\|T_1XD\|_2\left\|B^{\mathrm{T}}P\left[\sum_{j\in\mathcal{N}_i}a_{ij}(x_i - x_j) + a_{i0}(x_i - Xz)\right]\right\|_1
\end{aligned}$$

$$\begin{aligned}
&\bar{z}^{\mathrm{T}}[H \otimes (S + S^{\mathrm{T}}) - 2\mu H^2 \otimes (I)]\bar{z}^{\mathrm{T}} + 2\bar{z}^{\mathrm{T}}(H \otimes I)[q - \mathbf{1} \otimes (Dv)] \\
&\quad \le \bar{z}^{\mathrm{T}}[H \otimes (S + S^{\mathrm{T}}) - 2\mu H^2 \otimes (I)]\bar{z}^{\mathrm{T}} + 2\bar{z}^{\mathrm{T}}(H \otimes I)[-\gamma(I \otimes D)\,\mathrm{sgn}[(H \otimes D^{\mathrm{T}})\bar{z}] - \mathbf{1} \otimes (Dv)] \\
&\qquad + 2\sum_i(\gamma - \theta_i)\left\|D^{\mathrm{T}}\left[\sum_{j\in\mathcal{N}_i}a_{ij}(z_i - z_j) + a_{i0}(z_i - z)\right]\right\|_1 \\
&\quad \le \bar{z}^{\mathrm{T}}[H \otimes (S + S^{\mathrm{T}}) - 2\mu H^2 \otimes (I)]\bar{z}^{\mathrm{T}} + 2\sum_i(\gamma - \theta_i)\left\|D^{\mathrm{T}}\left[\sum_{j\in\mathcal{N}_i}a_{ij}(z_i - z_j) + a_{i0}(z_i - z)\right]\right\|_1
\end{aligned}$$

$$\begin{aligned}
\dot{V} &\le -\bar{x}^{\mathrm{T}}(H \otimes Q)\bar{x} + 2\sum_i(\gamma - \theta_i)\|T_1XD\|_2\left\|B^{\mathrm{T}}P\left[\sum_{j\in\mathcal{N}_i}a_{ij}(x_i - x_j) + a_{i0}(x_i - Xz)\right]\right\|_1 \\
&\quad + \bar{z}^{\mathrm{T}}[H \otimes (S + S^{\mathrm{T}}) - 2\mu H^2 \otimes (I)]\bar{z}^{\mathrm{T}} + 2\sum_i(\gamma - \theta_i)\left\|D^{\mathrm{T}}\left[\sum_{j\in\mathcal{N}_i}a_{ij}(z_i - z_j) + a_{i0}(z_i - z)\right]\right\|_1 \\
&\quad + 2\bar{x}^{\mathrm{T}}(H \otimes PBK_2)\bar{z} + 2\sum(\theta_i - \alpha)\dot{\theta}_i \\
&\le -\bar{x}^{\mathrm{T}}(H \otimes Q)\bar{x} + 2\sum_i(\alpha - \theta_i)\|T_1XD\|_2\left\|B^{\mathrm{T}}P\left[\sum_{j\in\mathcal{N}_i}a_{ij}(x_i - x_j) + a_{i0}(x_i - Xz)\right]\right\|_1 \\
&\quad + \bar{z}^{\mathrm{T}}[H \otimes (S + S^{\mathrm{T}}) - 2\mu H^2 \otimes (I)]\bar{z}^{\mathrm{T}} + 2\sum_i(\alpha - \theta_i)\left\|D^{\mathrm{T}}\left[\sum_{j\in\mathcal{N}_i}a_{ij}(z_i - z_j) + a_{i0}(z_i - z)\right]\right\|_1 \\
&\quad + 2\|(H \otimes PBK_2)\|_2\|\bar{x}^{\mathrm{T}}\|_2\|\bar{z}\|_2 + 2\sum(\theta_i - \alpha)\dot{\theta}_i \\
&= -\bar{x}^{\mathrm{T}}(H \otimes Q)\bar{x} + \bar{z}^{\mathrm{T}}[H \otimes (S + S^{\mathrm{T}}) - 2\mu H^2 \otimes (I)]\bar{z}^{\mathrm{T}} + 2\|(H \otimes PBK_2)\|_2\|\bar{x}^{\mathrm{T}}\|_2\|\bar{z}\|_2
\end{aligned}$$

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where $W(\bar{x}, \bar{z})$ is positive definite. Here $\bar{\lambda}$ is the minimal eigenvalue of all H and $\sigma_{\min}(Q)$ is the minimal eigenvalue of Q.

Hence, the closed-loop system is stable. Furthermore, due to the dynamics of $\bar{x}$, $\dot{\bar{x}}$ and $\bar{z}$ are bounded, so is $\dot{W}$. Consequently, W is uniformy continuous.

Obviously, V is decreasing and bounded in below by zero. As a result, $\lim_{t\to\infty} V(\bar{x}(t))$ exists. By the standard comparison theorem, we obtain

$$\int_{t_0}^{\infty} W(\bar{x}(t))\,\mathrm{d}t \le V(t_0) - V(\infty)$$

Recalling Barbalat's lemma, $W(\bar{x}) \to 0$ when $t \to \infty$, and hence $\bar{x}$, $\bar{z}$ and regulated variable e_i converges to be zero when time goes to infinity, while θ_i converges to be some finite value. □

Remark 5: Note that, no upper bound of the uncertainties is needed in protocols given in Theorem 2. The adaptive controller given in [12] is a special case of our controller, and dynamic gain techniques make it possible to solve this problem without using the upper bound of uncertainties.

Remark 6: A robustness issue may corrupt the controller's applications if there are disturbances and noises in the measurements, although it is theoretically proved to drive all regulated variables to vanish as $t \to \infty$. To tackle this problem, we propose an adaptive updated law with σ-modification [17] as follows (see (10)).

From the proof of Theorem 2, it can be easily verified that this control law will drive all regulated variables into a bounded set eventually. Furthermore, the bound can be small enough by tuning σ, which can be chosen according to practical control goals.

$$u_i = K_1 x_i + K_2 z_i - \theta_i \|T_1 XD\|_2 \mathrm{sgn}\left(B^{\mathrm{T}} P\left[\sum_{j\in\mathcal{N}_i} a_{ij}(x_i - x_j) + a_{i0}(x_i - Xz)\right]\right)$$
$$\dot{z}_i = Sz_i - \mu\left[\sum_{j\in\mathcal{N}_i} a_{ij}(z_i - z_j) + a_{i0}(z_i - z)\right] - \theta_i D\mathrm{sgn}\left(D^{\mathrm{T}}\left[\sum_{j\in\mathcal{N}_i} a_{ij}(z_i - z_j) + a_{i0}(z_i - z)\right]\right)$$
$$\dot{\theta}_i = \|T_1 XD\|_2 \left\| B^{\mathrm{T}} P\left[\sum_{j\in\mathcal{N}_i} a_{ij}(x_i - x_j) + a_{i0}(x_i - Xz)\right]\right\|_1 + \left\| D^{\mathrm{T}}\left[\sum_{j\in\mathcal{N}_i} a_{ij}(z_i - z_j) + a_{i0}(z_i - z)\right]\right\|_1 - \sigma\theta_i$$
$$\times\ (i = 1, \ldots, N) \qquad (10)$$

4 Discussions

It has been pointed out that our leader-following coordination problem can be seen as an extension of distributed output regulation, which covers many existing results. In this section, we provide special cases to show the relationship between ours and previous works.

(1) With distributed tracking: when $A = S = 0$, our problem is reduced to the tracking problem of single integrators if $B = D = 1$, or double-integrators [12] if $B = D = [0 \quad 1]^{\mathrm{T}}$. For general linear systems with $S = A$, $B = D$, $C = F$, it can be found Assumptions 3 and 4 always hold with $X = I$, $U = 0$ and $E = 0$, our control law solved the same problem considered in [13].

(2) With distributed output regulation: when the leader is unforced that is, $v = 0$, this problem has been investigated [3, 18, 19], and theoretical results have been achieved in the framework of distributed output regulation [9, 10] with consensus and synchronisation as its special cases. For example, in [20], the author considered a consensus problem, when there existed a leader, it is the case where $S = 0$, $D = 1$ and $v = 0$. However, those controllers fail to drive the regulated variables to be zero if $v \neq 0$ even knowing exactly v. The protocols proposed in the last section, firstly, can somehow solve the distributed output regulation problem as that in [10]. Secondly, the regulation variables can be driven to zero even under the uncertainties in the leader when the above assumptions hold.

(3) With other related works: when $N = 2$, we solve the generalised output regulation problem proposed in [16]. Furthermore, the condition (4) is relaxed than that given in [16]. At the same time, when the bound of uncertainties is unknown, the controller with an adaptive law enlarges the potential applications of this generalised output regulation problem.

In the following, we give a non-trivial class of systems that satisfy Assumptions 3 and 4. Hence, the leader-following coordination problem can be solved.

Consider the follower i as

$$\begin{aligned} \dot{x}_i &= Ax_i + BMz_i + Bu_i \\ \dot{z}_i &= (A_0 + B_0 M)z_i + Nx_i + B_0 u_i \\ y_i &= Cx_i \end{aligned} \qquad (11)$$

where (A, B) is controllable and A_0 is Hurwitz without common eigenvalues with A. The leader has the following dynamics

$$\begin{aligned} \dot{x}_0 &= Ax_0 + Bv \\ y_0 &= Cx_0 \end{aligned} \qquad (12)$$

Since A_0 has no common eigenvalues with A, there is X_2 such that $X_2 A = A_0 X_2 + N$. We can verify that $X = [I, X_2^{\mathrm{T}}]^{\mathrm{T}}$ and $U = -MX_2$ satisfy (3).

Clearly, $XB = [B, X_2 B]^{\mathrm{T}}$ cannot be decoupled from the output of composite system, since the necessary condition $[C, 0][B^{\mathrm{T}}, B^{\mathrm{T}} X_2^{\mathrm{T}}]^{\mathrm{T}} = CB = 0$ may not be satisfied. However, Assumption 4 holds with $K = [K_1, -M]$ and

$$E = \begin{bmatrix} I - BB^+ & 0 \\ -B_0B^+ & I \end{bmatrix}, \quad T_1 = [B^+, 0]$$

where K_1 is a matrix such that $A + BK_1$ is Hurwitz and B^+ is a generalised inverse of B such that $B^+B = I$.

5 Simulation

In this section, we provide an example to illustrate the effectiveness of our controllers.

Consider three followers in the following form

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = u_i \\ \dot{x}_{i3} = -x_{i3} + x_{i1} + u_i \\ y_i = x_{i1} \end{cases}$$

with the leader as

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = v \\ y_0 = z_1 \end{cases}$$

where $v = -z_1$.

Furthermore, we assume the interconnection topology is fixed satisfying Assumption 1 as follows: the interconnection topology is described by the following Laplacian

$$L = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and the diagonal matrices for the interconnection between followers and the agents are

$$A_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the followers have a different model from the leader, we can not use the controllers proposed in [13]; furthermore, the second assumption in [16] fails because $XD = [0, 1, 1]^{\mathrm{T}}$. However, it is easy to find that this system is in the form of (11) and Assumption 3 hold with $X_2 = [1, 1]$ and $X = [I, X_2^{\mathrm{T}}]^{\mathrm{T}}, U = 0$. Hence, if we take

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

then we find that $EXD = [0, 0, 0]^{\mathrm{T}}$. Hence, Assumption 4 holds with $K = [-64, -16, 0]$.

Since v depends on the unknown initials of z and we employed the adaptive control law.

By solving the Lyapunov equation (8) in Matlab with $Q = I$, we obtain

$$P = \begin{bmatrix} 3.7137 & 0.2934 & -0.2901 \\ 0.2934 & 0.5372 & -0.4877 \\ -0.2901 & -0.4877 & 0.5000 \end{bmatrix}$$

Setting $\mu = 20$, $\sigma = 0.1$, all initials of followers are randomly generated in $[-5, 5] \times [-5, 5] \times [-5, 5]$, and the controller for agent i $(i = 1, 2, 3)$ can be given in form of (10). Simulation results are showed in Figs. 1 and 2.

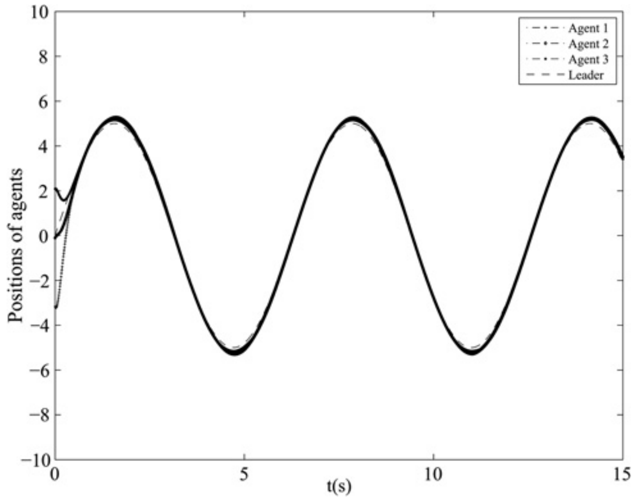

Fig. 1 *Tracking performance of agents*

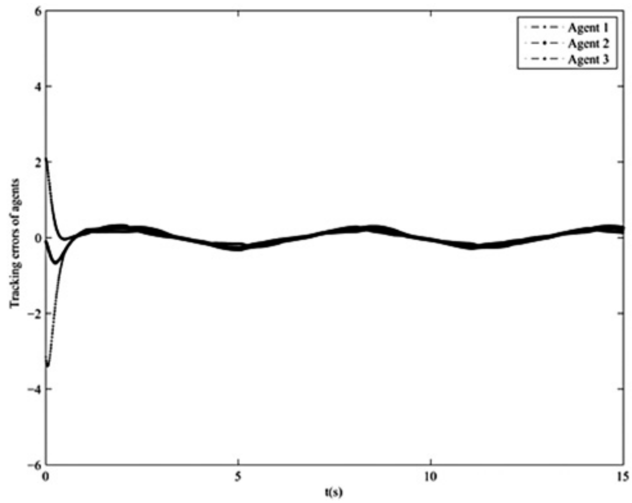

Fig. 2 *Tracking errors of agents*

6 Conclusions

A generalised leader-following coordination problem for multi-agent system was investigated where there are bounded unknown inputs in the leader dynamics. Two distributed control laws were given to solve this problem to deal with the cases with or without knowing the bound of uncertainty inputs in the leader. Simulation examples were also provided for illustration.

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