

Problem 1

The nonlinear dynamics equations for a single-link manipulator with flexible joints, damping ignored, is given by

$$\begin{aligned} I\ddot{q}_1 + MgL \sin q_1 + k(q_1 - q_2) &= 0 \\ J\ddot{q}_2 - k(q_1 - q_2) &= u \end{aligned}$$

where q_1 and q_2 are angular positions, I and J are moments of inertia, k is a spring constant, M is the total mass, L is a distance, and u is a torque input. Choose state variables for this system and write down the state equation.

Solution: Please write down your answer here.

Problem 2

Consider the system

$$\dot{x}_1 = 2x_1 - x_1x_2, \quad \dot{x}_2 = 2x_1^2 - x_2$$

- Find the equilibrium points and determine the type of each isolated one.
- Sketch the phase portrait of the system.

Solution: Please write down your answer here. Attach the code below if you use a computer program.

Problem 3

Consider the system

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 + \frac{1}{16}x_1^5 - x_2$$

- Find the equilibrium points and determine the type of each isolated one.
- Sketch the phase portrait of the system.

Solution: Please write down your answer here. Attach the code below if you use a computer program.

Problem 4

Consider the system

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 - \varepsilon x_1^2 x_2$$

What does its trajectory like near the origin? (Hints: Consider three different cases depending the sign of ε .)

Solution: Please write down your answer here.